TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 921

CONTRIBUTION TO THE AERODYNAMICS OF ROTATING-WING AIRCRAFT

By G. Sissingh

Luftfahrtforschung
Vol. 15, No. 6, June 6, 1938
Verlag von R. Oldenbourg, München und Berlin

Washington
December 1939

Straight Doc file

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 921

CONTRIBUTION TO THE AERODYNAMICS OF ROTATING-WING AIRCRAFT*

By G. Sissingh

The conventional calculating methods for rotors of rotating-wing aircraft with hinged blades are extended and refined.

The chief defect of the investigations up to now was the assumption of a more or less arbitrary "mean" drag coefficient for a section of the blade. This defect is remedied through replacement of the constant coefficient by a function of higher order which corresponds to the polar curve of the employed profile. In that way it is possible to extend the theory to include the entire range from "autogiro" without power input to the driven "helicopter" with forward-tilted rotor axis. The treatment includes the twisted rectangular blade and a nontwisted tapered blade.

The investigation is based on Wheatley's report (reference 10), which in turn is an extension of the well-known studies of Glauert (references 1, 3, 5) and Lock (reference 2). In the interests of clarity, the principal points of view of these analyses will be briefly reviewed.

SUMMARY AND DISCUSSION

Proceeding from the air flow and stresses on a section of the blade, the formulas for torque, axial and normal thrust of a linearly twisted rectangular blade, and a nontwisted tapered blade, are derived.

The principal advantage of the calculation over earlier investigations is that it is not necessary to depend on a rough estimation of the "mean" drag coefficient when

---

*"Beitrag zur Aerodynamik der Drehflügelflugzeuge." Luftfahrtforschung, vol. 15, no. 6, June 6, 1938, pp. 290-302.

Thesis submitted in partial fulfillment of the requirements for the degree of Engineer in Mechanical Engineering Aeronautics, Technische Hochschule Hannover.

computing a certain rotor as the polar coordinates of the blade section in question can be applied direct, independent of any assumptions or empirically obtained constants.

The theory includes all flight stages of rotating-wing aircraft from autogiro without power input to helicopter with forward-tilted rotor axis without regular propeller, and gives results up to coefficients of advance of from 0.4 to 0.5 which are in satisfactory agreement with wind-tunnel tests.

The calculation becomes uncertain when, at higher coefficients of advance, the zone of the separated flow within the rotor area is no longer covered by the stipulated assumptions and the effect of the occasional reversed-velocity region, for which the substitute functions for the air force coefficients on the blade element are no longer applicable, is observed as interference.

The agreement of the calculation with the experimental results will be proved and the effect of twist and taper on the quality of a rotor discussed in a future article.

I. NOTATION

1. Rotor

F	m^2,	swept-disk area of rotor.
R	m,	rotor radius.
z	–	number of blades.
t	m,	blade chord $= t_o\ (1 + p\ x)$.
t_o	m,	blade chord for $r = 0$, if the blade form is lengthened as far as the axis of rotation.
p	–	taper factor for a tapered blade.
σ	–	solidity $= \frac{z t_o}{\pi R}$.
γ	–	blade mass constant $= \frac{c_a'\ t_o\ R^4\ \rho}{I}$.

I mkg s^2, mass moment of inertia of rotor blade

$$I = \frac{1}{g} \int_{eR}^{R} P_{(r)}\ r\ (r - e\ R)\ dr$$

For $e = 0$, I is identical with the mass moment of inertia of the rotor blade about the horizontal hinge.

$P_{(r)}$ kg/m, blade weight per unit of length as $f(r)$.

M_G mkg, blade-weight moment referred to flapping hinge.

2. Nondimensional Distances Referred to Radius

$x = r/R$ - distance of a blade section at distance r from the axis of rotation.

e - distance of flapping hinge from rotor axis.

B - factor allowing for thrust decrease at blade tip for finite blade number,

$$B \cong 1 - \frac{t_o\ (1+0.7\ p)}{1.5\ R}.$$

See TM 990 p.8 for derivation of $B = 1 - \frac{\sqrt{k_{sa}}}{z}$

b - factor allowing for separation flow on returning blade at higher coefficients of advance.

3. Rotor Coefficients

k_{sa} - $2C_T$ axial thrust } propeller-axis system.
k_{sn} - normal thrust

k_{sv} - vertical thrust } wind-axis system.
k_{sh} - horizontal thrust

k_d - $2C_Q$ torque -
positive for driven rotor,
negative when operating as windmill;

forces = coefficient F $U^2\ \rho/2$,

moment = coefficient FR $U^2\ \rho/2$,

ρ air density.

4. Speeds

u m/s, tip of blade section at distance x.

U m/s, tip speed at tip area.

V m/s, flying speed.

v_d m/s, axial flow velocity in the normal plane;
$v_d = V \sin \alpha - w$, positive if flow is upward.

w m/s, rotor induced velocity.

v m/s, resultant relative inflow at a blade section with component;
$U\,v_x$ in the normal plane,
$U\,v_y$ perpendicular to the normal plane.

ω s^{-1}, rotor angular velocity.

μ - coefficient of advance.

λ_d - axial flow $\lambda_d = v_d/U$.

5. Force Coefficients

c_a - lift coefficient,
$c_a = c_a' \alpha_p$.

c_a' - lift gradient $\cong 5.6$.

c_{ao} - constant lift coefficient for separated flow zones.

c_w - drag coefficient

$$c_w = i\,(c_o + c_1\,\alpha_p + c_2\,\alpha_p^2).$$

c_o, c_1, c_2 profile constants.

i - factor allowing for shifting of blade at higher coefficients of advance; $i = 1$ for small coefficients of advance.

c_t – coefficient for tangential force

$$c_t = c_w \cos\varphi - c_a \sin\varphi.$$

For computing c_a' and c_o to c_2 from the profile polar, use an aspect ratio of

$$\frac{t_o\,(1 + 0.7\,p)}{1.5\,R}$$

as basis.

6. Angles

α – rotor angle of attack (normal plane) to flow.

ϑ – blade angle of attack, aerodynanic setting of a blade section to normal plane;

$$\vartheta = \vartheta_o + x\,\vartheta_1.$$

φ – angle of flow, angle of blade inflow to normal plane; $\tan\varphi = v_y/v_x$.

α_p – operating angle of a blade section at $\alpha_p = \varphi + \vartheta$.

ψ – blade angle of rotation, neutral position back.

β – flapping angle referred to normal plane

$$\beta = a_o - a_1 \cos\psi - b_1 \sin\psi.$$

7. Abbreviations

$$\zeta^n = b^n - \mu^n$$

$$\iint f(x,\psi)\,dx\,d\psi = \int_0^1\int_0^{2\pi} f(x,\psi)\,dx\,d\psi - 2\int_0^{-\mu\sin\psi}\int_\pi^{2\pi} f(x,\psi)\,dx\,d\psi$$

$$\iint^{x} f(x,\psi)\,dx\,d\psi = \int_0^B\int_0^{2\pi} f(x,\psi)\,dx\,d\psi - 2\int_0^{-\mu\sin\psi}\int_\pi^{2\pi} f(x,\psi)\,dx\,d\psi.$$

II. INTRODUCTION

1. Flow through Rotor

If the normal plane of a rotor faces the air flow at an angle α, the axial flow through the rotor is, according to figure 1:

$$v_d = V \sin \alpha - w$$

where w is the rotor induced velocity which for coefficients of advance of around $\mu \geqq 0.2$ within the swept rotor-disk area may be considered as being constant. It is usually figured at half the amount of that computed by the momentum theory for flow at infinity behind the rotor.

According to Küssner's hypothesis (reference 28), the comprised air quantity is the quantity of air passing through a sphere circumscribing the rotor. On these premises the rotor angle of attack α is:

$$\tan \alpha = \frac{\lambda_d}{\mu} + \frac{k_{sa}}{4\mu\sqrt{\mu^2 + \lambda_d^2}} \tag{1}$$

The assumption of constant axial flow within the normal plane is no longer justified for the hovering stage. In this case the actual conditions are better represented by a distribution of the induced velocity increasing linearly with the radius. Thereby it can be assumed that the mean value w from momentum theory is reached by the particular blade elements at distance 0.7 R. (See Isacco, reference 19.)

2. Flow on a Section of the Blade

The equations for the components of the relative flow of a blade section at distance x are:

$U v_x = U (x + \mu \sin \psi)$ in normal plane direction (2)

$U v_y = U \left(\lambda_d - x \frac{d\beta}{d\psi} - \mu \beta \cos \psi\right)$ perpendicular to the normal plane (fig. 2) (3)

Radial velocities are temporarily disregarded. In equation (3) β is the flapping angle in relation to the normal plane. Its representation is limited to a Fourier series of first order of the form:

$$\beta = a_o - a_1 \cos \psi - b_1 \sin \psi \tag{4}$$

since the calculation of different examples proved that the terms of higher order had practically no effect on the rotor quantities in question.

3. Air-Force Coefficients at Blade Section Expressed by Substitute Functions

The significant forces on a section of the blade are the lift, the drag, and the tangential force in circumferential direction. In compliance with modern autogiro practice, the center of pressure of the air force is made to coincide with the centroid of the blade. As a result, the blades are, while revolving, not subject to period distortions conditioned by changing air-force moments.

The lift coefficient is, as usual, expressed by

$C_\ell \quad a \quad \alpha_p$

$$c_a = c_a{}' \alpha_p \tag{5}$$

The lift gradient $c_a{}'$ for normal airfoil sections may be put at 5.6. Deviating from previous investigations, the drag coefficient is replaced by the following function of the operating angle

$$c_w = c_o + c_1 \alpha_p + c_2 \alpha_p{}^2 \tag{6}$$

The tangential force coefficient is

coeff long force

$$c_t = c_w \cos \varphi - c_a \sin \varphi$$

In view of the smallness of φ, c_t can be expressed sufficiently exact with

$$c_t = c_w - c_a \varphi \tag{7}$$

The sign - in accordance with that for the torque - is herewith so chosen that a section of the blade actuated by the air forces (autorotation range) corresponds to a negative c_t or k_d, respectively.

With $\varphi = \alpha_p - \vartheta$ and insertion of equations (5) and (6) in equation (7), the substitute function for the tangential force coefficient follows as

$$c_t = c_o + \alpha_p \left(c_1 + \vartheta\ c_a{}'\right) + \alpha_p{}^2 \left(c_2 - c_a{}'\right) \tag{8}$$

A comparison of the substitute functions with the actual variation of the particular coefficient indicates that c_w is very well represented within the range of actual practice. For c_a, on the other hand, the breakaway of the flow is not taken into account. Consequently, the substitute function for c_t holds only for operating angles up to $\alpha_p \cong 15^\circ$.

A survey over the fundamental course of c_t is obtained from figure 3, where c_t with its substitute function has been plotted against α_p for various blade angles of attack ϑ.

The curves disclose the fact known from Schrenk's presentation (reference 24) that the α_p range of propelling air forces decreases with increasing blade angle of attack ϑ until finally no propulsion is at all possible on the particular profile at $\vartheta \cong 13^\circ$.

They also confirm, as previously mentioned, the good agreement obtaining between the substitute function and the actual course up to $\alpha_p \cong 15^\circ$. Then, of course, the representation is fundamentally erroneous.

However, since in normal horizontal flight the central and outer blade elements are struck at small operating angles, the representation of the coefficients through the chosen substitute functions is admissible. Where, particularly at high coefficients of advance, the assumptions are no longer complied with, correction factors are employed.*

---

* The substitution of c_a and c_w with functions of higher order is fairly obvious. But then complications occur in the subsequent integrations according to the study.

Reversed flow.- Conformable to Wheatley (reference 10), the lift in the reversed-velocity region is written in equation (5) as

$$\alpha_p = -\frac{v_y}{v_x} - \vartheta$$

which gives for c_a

$$c_a = c_a' \left(-\frac{v_y}{v_x} - \vartheta\right) \tag{9}$$

For c_w we introduce in equation (6):

$$\alpha_p = -\frac{v_y}{v_x} + \vartheta^*$$

Herewith

$$c_w = c_o + c_1\left(-\frac{v_y}{v_x} + \vartheta\right) + c_2\left(-\frac{v_y}{v_x} + \vartheta\right)^2 \tag{10}$$

III. HOVERING STAGE

Axial flow and velocities at a section of the blade.- According to elementary jet theory, the mean axial flow is:

$$\lambda_{d_{mean}} = -\frac{\sqrt{k_{sa}}}{2}$$

---

*The assumption $\alpha_p = -v_y/v_x + \vartheta$ deviating from equation (9) for the drag coefficient, affords several simplifications in the subsequent calculation. Since the drag in reverse flow ($\alpha_p \cong 180^o$) is higher than in flow from the leading to the trailing edge, c_w would have to be replaced by a function with other coefficients c_o to c_2. But the coefficients of advance $\mu < 0.5$ in question do not justify this action, since the effect of the reversed-velocity region on the forces and moments of the whole rotor is quite small. A certain balance is established by the fact that the expression $-v_y/v_x$ is, in most cases, positive, and then a greater operating angle - that is, higher drag coefficient, is involved.

Assuming a linear distribution of the rotor-induced velocity over the radius, whereby the above $\lambda_{d_{mean}}$ is reached at $x = 0.7$, it is:

$$v_y = x \lambda_{d_1}$$

with

$$\lambda_{d_1} = - \frac{\sqrt{k_{sa}}}{1.4} \tag{11}$$

Since $v_x = x$ for $\mu = 0$, the angle of flow φ (fig. 2) is equal to λ_{d_1} independent of the position of the section of the blade.

Axial thrust.- A z-blade rotor with blade chord t has an axial thrust of

no. blades velocity density chord lift coeff

$$z \int_0^{BR} v^2 \frac{\rho}{2} t c_a dr$$

The factor B allows for the thrust decrease at the blade tip, due to the finite blade number. Substitution in the conventional manner of the component $U v_x$ in the normal plane for the resultant inflow velocity v, gives the axial thrust coefficient

$$k_{sa} = \sigma \int_0^B v_x^2 c_a dx$$

Expressing the blade form and twist by functions:

$$t = t_o (1 + p x)$$

$$\vartheta = \vartheta_o + x \vartheta_1$$

affords

$$k_{sa} = \sigma c_a' \left[\int_0^B x^2 (\lambda_{d_1} + \vartheta_o + x \vartheta_1) dx + p \int_0^B x^3 (\lambda_{d_1} + \vartheta_o + x \vartheta_1) dx \right] \tag{12}$$

The solution is:

$$k_{sa} = \sigma\, c_a{}' \left[\frac{1}{3} B^3 \lambda_{d_1} + \frac{1}{3} B^3 \vartheta_o + \frac{1}{4} B^4 \vartheta_1 + p\left(\frac{1}{4} B^4 \vartheta_o + \frac{1}{4} B^4 \lambda_{d_1} + \frac{1}{5} B^5 \vartheta_1\right)\right] \qquad (13)$$

<u>Torque</u>.- In accordance with the preceding considerations, it is:

$$\begin{aligned} k_d = \sigma \Bigg[& \int_0^1 v_x{}^2\, x\, c_w\, dx + p \int_0^1 v_x{}^2\, x^2\, c_w\, dx \\ & - c_a{}' \int_0^B v_x{}^2\, x\, \frac{v_y}{v_x}\left(\frac{v_y}{v_x} + \vartheta_o + x\,\vartheta_1\right) dx \\ & - p\, c_a{}' \int_0^B v_x{}^2\, x^2\, \frac{v_y}{v_x}\left(\frac{v_y}{v_x} + \vartheta_o + x\,\vartheta_1\right) dx \Bigg] \end{aligned}$$

Introducing c_w according to equation (6) gives, with $v_x = x$ and $v_y/v_x = \lambda_{d_1}$:

$$\begin{aligned} \frac{k_d}{\sigma} = & \int_0^1 x^3 \left\{ c_o + c_1 (\lambda_{d_1} + \vartheta_o + x\,\vartheta_1) + c_2 (\lambda_{d_1} + \vartheta_o + x\,\vartheta_1)^2 \right\} dx \\ & + p \int_0^1 x^4 \left\{ c_o + c_1 (\lambda_{d_1} + \vartheta_o + x\,\vartheta_1) + c_2 (\lambda_{d_1} + \vartheta_o + x\,\vartheta_1)^2 \right\} dx \\ & - c_a{}' \int_0^B x^3\, \lambda_{d_1} (\lambda_{d_1} + \vartheta_o + x\,\vartheta_1)\, dx \\ & - p\, c_a{}' \int_0^B x^4\, \lambda_{d_1} (\lambda_{d_1} + \vartheta_o + x\,\vartheta_1)\, dx \end{aligned}$$

The result is:

$$k_d = \sigma \Big[\frac{1}{4} c_0 + c_1 \left(\frac{1}{4} \lambda_{d_1} + \frac{1}{4} \vartheta_0 + \frac{1}{5} \vartheta_1 \right)$$

$$+ c_2 \left(\frac{1}{4} \lambda_{d_1}^2 + \frac{1}{2} \lambda_{d_1} \vartheta_0 + \frac{2}{5} \lambda_{d_1} \vartheta_1 \right.$$

$$\left. + \frac{1}{4} \vartheta_0^2 + \frac{1}{6} \vartheta_1^2 + \frac{2}{5} \vartheta_0 \vartheta_1 \right)$$

$$+ p \left\{ \frac{1}{5} c_0 + c_1 \left(\frac{1}{5} \lambda_{d_1} + \frac{1}{5} \vartheta_0 + \frac{1}{6} \vartheta_1 \right) \right.$$

$$+ c_2 \left(\frac{1}{5} \lambda_{d_1}^2 + \frac{2}{5} \lambda_{d_1} \vartheta_0 + \frac{1}{3} \lambda_{d_1} \vartheta_1 \right.$$

$$\left. \left. + \frac{1}{5} \vartheta_0^2 + \frac{1}{7} \vartheta_1^2 + \frac{1}{3} \vartheta_0 \vartheta_1 \right) \right\}$$

$$- c_a' \left\{ \frac{1}{4} B^4 \lambda_{d_1}^2 + \lambda_{d_1} \left(\frac{1}{4} B^4 \vartheta_0 + \frac{1}{5} B^5 \vartheta_1 \right) \right\}$$

$$- p\, c_a' \left\{ \frac{1}{5} B^5 \lambda_{d_1}^2 + \lambda_{d_1} \left(\frac{1}{5} B^5 \vartheta_0 + \frac{1}{6} B^6 \vartheta_1 \right) \right\} \Big] \qquad (14)$$

IV. HORIZONTAL FLIGHT

Following the study of hovering the true flight stages will be treated; that is, those stages in which the rotor flow is approximately in direction of the normal plane. They are largely the operating stages of normal horizontal flight.

The limit toward one side is formed by the autogiro without power input (free-wheeling condition with $k_d = 0$), and on the other side, by the driven helicopter without regular propeller but forward-tilted rotor axis. Between these two limits lie the numerous possibilities of a "helicogiro," on which the power can be arbitrarily distributed over rotor and regular propeller.

The study of the twisted rectangular blade and the nontwisted tapered blade is carried out separately. Torque, axial and normal thrust are computed. The change-over from these forces to the flow-axis system (lift and drag) is effected by simple transformation of the coordinates (fig. 4).

To characterize the particular blade, the coefficients of axial and normal thrust and torque are supplemented by the indices v and t for twist and taper, respectively.

1. Linearly Twisted Rectangular Blade

The equations for the coefficients of flapping motion and axial thrust are taken from Wheatley's report (reference 10) which, at a twist of the form

$$\vartheta = \vartheta_o + x\,\vartheta_1$$

read as follows:

$$a_o = \frac{\gamma}{2}\left\{\frac{1}{3}B^3\lambda_d + 0.08\,\mu^3\lambda_d + \frac{1}{4}\vartheta_o\left(B^4 + \mu^2 B^2 - \frac{1}{8}\mu^4\right) + \frac{1}{5}\vartheta_1\left(B^5 + \frac{5}{6}\mu^2 B^3\right)\right\} - \frac{M_G}{I\,\omega^2} \tag{15}$$

$$a_1 = \frac{2\mu}{B^4 - \frac{1}{2}\mu^2 B^2} \times \left\{\lambda_d\left(B^2 - \frac{1}{4}\mu^2\right) + \vartheta_o\left(\frac{4}{3}B^3 + 0.106\,\mu^3\right) + \vartheta_1 B^4\right\} \tag{16}$$

$$b_1 = \frac{4\mu B}{B^2 + \frac{1}{2}\mu^2}\,a_o\left\{\frac{1}{3} + \frac{0.035}{B^3}\mu^3\right\} \tag{17}$$

$$k_{sav} = \sigma\,c_a'\left\{\frac{1}{2}\lambda_d\left(B^2 + \frac{1}{2}\mu^2\right) + \vartheta_o\left(\frac{1}{3}B^3 + \frac{1}{2}\mu^2 B - \frac{4}{9\pi}\mu^3\right) + \frac{1}{8}\mu^3 a_1 + \vartheta_1\left(\frac{1}{4}B^4 + \frac{1}{4}\mu^2 B^2 - \frac{1}{32}\mu^4\right)\right\} \tag{18}$$

The blade constant γ for a_0 in equation (15) is a measure for the ratio of the air-force moments to the mass-force moments:

$$\gamma = \frac{t_0 \, c_a' \, R^4 \, \rho}{I}$$

with

$$I = \frac{1}{g} \int_{eR}^{R} P_{(r)} \, r \, (r - e\,R) \, dr \qquad (19)$$

For the first cases of $e = 0$ (flapping hinge in axis of rotation) I is identical with the mass moment of inertia of a blade with respect to the rotor axis

$$I_{e=o} = \frac{1}{g} \int_{o}^{R} P_{(r)} \, r^2 \, dr$$

Here P is the blade weight per unit length. Assuming P = constant (uniform weight distribution across the blade length), the expression for γ further reduces to

$$\gamma = 3 \, g \, \rho \, c_a' \, \frac{R \, t_0}{P}$$

With the normal values:

$$\rho = 0.125 \text{ kg} \cdot \text{s}^2 \text{ m}^{-4}$$

$$g = 9.81 \text{ m/s}^2$$

$$c_a' = 5.6$$

it finally gives for this specific case ($e = 0$, P = constant)

$$\gamma = 20.6 \, \frac{R \, t_0}{P} \qquad (20)$$

Calculation of torque,

$$M_d = \frac{z \cdot t_0 \cdot \varrho/2}{2\pi}\Bigg[\int_0^R \int_0^{2\pi} v^2 \cdot r \cdot c_w \cdot \cos\varphi \, dr \, d\psi$$

$$- c_a' \Bigg\{ + \int_0^{BR} \int_0^{\pi} v^2 \cdot r \cdot \frac{v_y}{v_x} \cdot \left(\frac{v_y}{v_x} + \vartheta_0 + x\,\vartheta_1\right) dr\, d\psi$$

$$+ \int_{-\mu R \cdot \sin\psi}^{BR} \int_{\pi}^{2\pi} v^2 \cdot r \cdot \frac{v_y}{v_x} \left(\frac{v_y}{v_x} + \vartheta_0 + x\,\vartheta_1\right) dr\, d\psi$$

$$- \int_0^{-\mu R \cdot \sin\psi} \int_{\pi}^{2\pi} v^2 \cdot r \left(-\frac{v_y}{v_x}\right)\left(-\frac{v_y}{v_x} - \vartheta_0 - x\,\vartheta_1\right) dr\, d\psi \Bigg\}\Bigg] \quad \ldots\ldots\ldots (21)$$

The lift contribution having been investigated by Wheatley (reference 10), the terms with c_w must be evaluated. They give:

$$k_{d\ c_w} = \frac{\sigma}{2\pi}\Bigg[\int_0^1 \int_0^{\pi} v_x^2 \cdot x \cdot c_w \cdot dx \cdot d\psi$$

$$+ \int_{-\mu\sin\psi}^{1} \int_{\pi}^{2\pi} v_x^2 \cdot x \cdot c_w \cdot dx \cdot d\psi$$

$$- \int_0^{-\mu\sin\psi} \int_{\pi}^{2\pi} v_x^2 \cdot x \cdot c_w \cdot dx \cdot d\psi\Bigg] \quad \ldots\ldots\ldots (22)$$

$$= \frac{\sigma}{2\pi}\Bigg[\int_0^1 \int_0^{\pi} v_x^2 \cdot x \Bigg\{c_0 + c_1\left(\frac{v_y}{v_x} + \vartheta_0 + x\,\vartheta_1\right) + c_2\left(\frac{v_y}{v_x} + \vartheta_0 + x\,\vartheta_1\right)^2\Bigg\} dx\, d\psi$$

$$+ \int_{-\mu\sin\psi}^{1} \int_{\pi}^{2\pi} v_x^2 \cdot x \Bigg\{c_0 + c_1\left(\frac{v_y}{v_x} + \vartheta_0 + x\,\vartheta_1\right) + c_2\left(\frac{v_y}{v_x} + \vartheta_0 + x\,\vartheta_1\right)^2\Bigg\} dx\, d\psi$$

$$- \int_0^{-\mu\sin\psi} \int_{\pi}^{2\pi} v_x^2 \cdot x \Bigg\{c_0 + c_1\left(-\frac{v_y}{v_x} + \vartheta_0 + x\,\vartheta_1\right) + c_2\left(-\frac{v_y}{v_x} + \vartheta_0 + x\,\vartheta_1\right)^2\Bigg\} dx\, d\psi\Bigg] \,. \quad (23)$$

$$
\begin{aligned}
= \frac{\sigma}{2\pi} \Big[& (c_0 + c_1 \cdot \vartheta_0 + c_2 \cdot \vartheta_0^2) \int\int v_x^2 \cdot x \cdot dx\, d\psi \\
& + (c_1 \vartheta_1 + 2\, \vartheta_0\, \vartheta_1\, c_2) \int\int v_x^2 \cdot x^2 \cdot dx\, d\psi \\
& + c_2 \cdot \vartheta_1^2 \int\int v_x^2 \cdot x^3 \cdot dx\, d\psi \\
& + c_2 \int\int v_y^2 \cdot x \cdot dx\, d\psi \\
& + (c_1 + 2\, \vartheta_0\, c_2) \int_0^1\int_0^{2\pi} v_x \cdot v_y \cdot x \cdot dx\, d\psi \\
& + 2\, \vartheta_1 \cdot c_2 \int_0^1\int_0^{2\pi} v_x \cdot v_y \cdot x^2 \cdot dx\, d\psi \Big] \quad (24)
\end{aligned}
$$

After arrangement of the terms, together with the contribution of c_a taken from Wheatley's report, the torque of the linearly twisted rectangular blade is:

$$
\begin{aligned}
\frac{k_{dv}}{\sigma} = & (c_0 + c_1 \cdot \vartheta_0 + c_2 \vartheta_0^2) \left(\frac{1}{4} + \frac{1}{4}\mu^2 - \frac{1}{32}\mu^4\right) \\
& + \frac{1}{3}\lambda_d\, (c_1 + 2\, \vartheta_0 \cdot c_2) + (\vartheta_1 \cdot c_1 + 2\, \vartheta_0\, \vartheta_1 \cdot c_2) \left(\frac{1}{5} + \frac{1}{6}\mu^2\right) \\
& + \vartheta_1^2 \cdot c_2 \left(\frac{1}{6} + \frac{1}{8}\mu^2\right) + \frac{1}{2}\lambda_d \cdot \vartheta_1 \cdot c_2 + c_2 \Big\{ a_1^2 \left(\frac{1}{8} + \frac{3}{16}\mu^2\right) \\
& + \mu\, \lambda_d\, a_1 \left(\frac{1}{2} - \frac{3}{8}\mu^2\right) + \lambda_d^2 \left(\frac{1}{2} - \frac{1}{4}\mu^2\right) - \frac{1}{3}\mu\, a_0\, b_1 \\
& + a_0^2 \left(\frac{1}{4}\mu^2 - \frac{1}{16}\mu^4\right) + b_1^2 \left(\frac{1}{8} + \frac{1}{16}\mu^2\right) \Big\} \\
& - c_a' \Big\{ \lambda_d^2 \left(\frac{1}{2} B^2 - \frac{1}{4}\mu^2\right) + \lambda_d \Big(\frac{1}{3} B^3\, \vartheta_0 + \frac{2}{9\pi}\mu^3\, \vartheta_0 \\
& + \frac{1}{4} B^4\, \vartheta_1 + \frac{1}{32}\mu^4 \vartheta_1\Big) + \mu\, \lambda_d\, a_1 \left(\frac{1}{2} B^2 - \frac{3}{8}\mu^2\right) \\
& + a_0^2 \left(\frac{1}{4}\mu^2 B^2 - \frac{1}{16}\mu^4\right) - \frac{1}{3}\mu\, a_0\, b_1 \cdot B^3 \\
& + a_1^2 \left(\frac{1}{8} B^4 + \frac{3}{16}\mu^2 B^2\right) + b_1^2 \left(\frac{1}{8} B^4 + \frac{1}{16}\mu^2 B^2\right) \Big\} \;. \quad (25)
\end{aligned}
$$

<u>Normal thrust</u>.- The normal thrust is built up from the contributions of the forces at the blade in circumferential direction (normal plane) and the lift components created by the flapping motion.

In the normal plane a section of the blade at distance x has at blade setting ψ a normal thrust of

$$v^2 \frac{\rho}{2}\, t\, dr\, c_t \sin\psi$$

$$= v^2 \frac{\rho}{2}\, t\, dr \sin\psi\, (c_w - c_a \sin\varphi)$$

Considering first the share of c_w, we find:

$$
\begin{aligned}
k_{sn\,\mathrm{I}} = \frac{\sigma}{2\pi} \Big[& \int_0^1\int_0^{\pi} v_x^2 \cdot c_w \cdot \sin\psi \cdot dx\, d\psi \\
& + \int_{-\mu\sin\psi}^{1}\int_{\pi}^{2\pi} v_x^2 \cdot c_w \cdot \sin\psi \cdot dx\, d\psi \\
& - \int_0^{-\mu\sin\psi}\int_{\pi}^{2\pi} v_x^2 \cdot c_w \cdot \sin\psi \cdot dx \cdot d\psi \Big] \quad . \; . \; . \; . \; . \quad (26)
\end{aligned}
$$

These expressions outwardly have a resemblance to equations (22) to (24). There sin ψ substitutes for 1 x x and accordingly, we have:

$$k_{sn\,\mathrm{I}} = \frac{\sigma}{2\pi}\Big[(c_0 + c_1\cdot\vartheta_0 + c_2\cdot\vartheta_0^2)\int\int v_x^2\cdot\sin\psi\,dx\,d\psi$$
$$+ (c_1\cdot\vartheta_1 + 2\vartheta_0\vartheta_1\cdot c_2)\int\int v_x^2\cdot x\cdot\sin\psi\,dx\,d\psi$$
$$+ c_2\cdot\vartheta_1^2\int\int v_x^2\cdot x^2\cdot\sin\psi\,dx\,d\psi$$
$$+ c_2\int\int v_y^2\cdot\sin\psi\,dx\,d\psi$$
$$+ (c_1 + 2\vartheta_0\cdot c_2)\int_0^1\int_0^{2\pi} v_x\cdot v_y\cdot\sin\psi\,dx\,d\psi$$
$$+ 2\vartheta_1\cdot c_2\int_0^1\int_0^{2\pi} v_x\cdot v_y\cdot x\cdot\sin\psi\,dx\,d\psi\Big] \quad . . . (27)$$

The share of c_a in the normal plane amounts to:

$$k_{sn\,\mathrm{II}} = -\frac{\sigma\cdot c_a'}{2\pi}\times$$
$$\Big[+\int_0^B\int_0^\pi v_x^2\cdot\frac{v_y}{v_x}\cdot\left(\frac{v_y}{v_x} + \vartheta_0 + x\vartheta_1\right)\sin\psi\,dx\,d\psi$$
$$+\int_{-u\sin\psi}^{B}\int_\pi^{2\pi} v_x^2\cdot\frac{v_y}{v_x}\left(\frac{v_y}{v_x} + \vartheta_0 + x\vartheta_1\right)\sin\psi\,dx\,d\psi$$
$$-\int_0^{-u\sin\psi}\int_\pi^{2\pi} v_x^2\cdot\left(-\frac{v_y}{v_x}\right)\left(-\frac{v_y}{v_x} - \vartheta_0 - x\vartheta_1\right)\sin\psi\,dx\,d\psi\Big] \quad . . . (28)$$

$$= -\frac{\sigma\,c_a'}{2\pi}\Big[\iint^{x} v_y^2\cdot\sin\psi\,dx\,d\psi$$
$$+\iint^{x}\vartheta_0\cdot v_x\cdot v_y\cdot\sin\psi\,dx\,d\psi$$
$$+\iint^{x}\vartheta_1\cdot v_x\cdot v_y\cdot x\cdot\sin\psi\,dx\,d\psi\Big] \quad . . . . (29)$$

The normal thrust of a blade with setting ψ is, for flapping angle β and lift A,

$$-A\,\beta\,\cos\psi$$

according to which:

$$k_{sn\,\mathrm{III}} = -\frac{\sigma\cdot c_a'}{2\pi}\Big[\int_0^B\int_0^\pi v_x^2\cdot\left(\frac{v_y}{v_x} + \vartheta_0 + x\vartheta_1\right)\beta\cdot\cos\psi\,dx\,d\psi$$
$$+\int_{-\mu\sin\psi}^{B}\int_\pi^{2\pi} v_x^2\left(\frac{v_y}{v_x} + \vartheta_0 + x\vartheta_1\right)\beta\cdot\cos\psi\cdot dx\,d\psi$$
$$+\int_0^{-\mu\sin\psi}\int_\pi^{2\pi} v_x^2\cdot\left(-\frac{v_y}{v_x} - \vartheta_0 - x\vartheta_1\right)\beta\cdot\cos\psi\,dx\,d\psi\Big]$$

$$= -\frac{\sigma\cdot c_a'\cdot\beta}{2\pi}\Big[\iint^{x} v_x\cdot v_y\cdot\cos\psi\,dx\,d\psi$$
$$+\vartheta_0\iint^{x} v_x^2\cdot\cos\psi\,dx\,d\psi$$
$$+\vartheta_1\iint^{x} v_x^2\cdot x\cdot\cos\psi\,dx\,d\psi\Big] \quad . . . . . (30)$$

Introducing $\beta = a_0 - a_1 \cos\psi - b_1 \sin\psi$, we find:

$$\begin{aligned}
k_{sn\,\mathrm{III}} = -\frac{\sigma \cdot c_a'}{2\pi} \cdot \Big[&+ a_0 \iint^{x} v_x \cdot v_y \cdot \cos\psi \, dx\, d\psi \\
&- a_1 \iint^{x} v_x \cdot v_y \cdot \cos^2\psi \, dx\, d\psi \\
&- b_1 \iint^{x} v_x \cdot v_y \cdot \cos\psi \cdot \sin\psi \cdot dx\, d\psi \\
&+ \vartheta_0 \cdot a_0 \iint^{x} v_x^2 \cdot \cos\psi \cdot dx\, d\psi \\
&- \vartheta_0 \cdot a_1 \iint^{x} v_x^2 \cdot \cos^2\psi \cdot dx\, d\psi \\
&- \vartheta_0 \cdot b_1 \iint^{x} v_x^2 \cdot \cos\psi \cdot \sin\psi \cdot dx\, d\psi \\
&+ \vartheta_1 \cdot a_0 \iint^{x} v_x^2 \cdot x \cdot \cos\psi \cdot dx\, d\psi \\
&- \vartheta_1 \cdot a_1 \iint^{x} v_x^2 \cdot x \cdot \cos^2\psi \cdot dx\, d\psi \\
&- \vartheta_1 \cdot b_1 \iint^{x} v_x^2 \cdot x \cdot \cos\psi \cdot \sin\psi \cdot dx\, d\psi \Big] \qquad (31)
\end{aligned}$$

The ultimate result is the sum of $k_{sn\,\mathrm{I}} \div k_{sn\,\mathrm{III}}$ as coefficient of normal thrust:

$$\begin{aligned}
k_{snv} = \sigma \Big[&(c_0 + c_1 \cdot \vartheta_0 + c_2 \cdot \vartheta_0^2)\left(\frac{1}{2}\mu + \frac{1}{8}\mu^3\right) \\
&+ (c_1 \cdot \vartheta_1 + 2\, c_2\, \vartheta_0\, \vartheta_1)\left(\frac{1}{3}\mu + \frac{4}{45\pi}\mu^4\right) \\
&+ \frac{1}{4}\mu\, c_2\, \vartheta_1^2 + 2\, c_2\, \vartheta_1 \left\{\frac{1}{4}\mu\, \lambda_d - a_1\left(\frac{1}{8} - \frac{1}{16}\mu^2\right)\right\} \\
&+ (2\, c_2\, \vartheta_0 + c_1)\left(\frac{1}{2}\mu\, \lambda_d - \frac{1}{6} a_1 + \frac{1}{8}\mu^2 a_1\right) \\
&+ c_2 \Big\{\frac{1}{2}\mu\, \lambda_d^2 + \frac{1}{8}\mu^3 a_0^2 - a_0\, b_1\left(\frac{1}{4}\mu^2 - \frac{4}{15\pi}\mu^3\right) \\
&\qquad - \lambda_d \cdot a_1\left(\frac{1}{2} - \frac{5}{8}\mu^2\right) - a_1^2\left(\frac{1}{8}\mu - \frac{11}{48}\mu^3\right) \\
&\qquad + b_1^2\left(\frac{1}{8}\mu + \frac{1}{48}\mu^3\right)\Big\} \\
&+ c_a' \Big\{+ a_1\, \lambda_d\left(\frac{3}{4}B^2 - \frac{9}{16}\mu^2\right) \\
&\qquad + a_1 \cdot \vartheta_0\left(\frac{1}{3}B^3 + \frac{10}{45\pi}\mu^3\right) \\
&\qquad + a_1 \cdot \vartheta_1\left(\frac{1}{4}B^4 + \frac{7}{192}\mu^4\right) \\
&\qquad + a_0^2\left(\frac{1}{4}B^2\mu - \frac{1}{16}\mu^3\right) \\
&\qquad + a_1^2\left(\frac{1}{4}B^2\mu - \frac{3}{16}\mu^3\right) \\
&\qquad - a_0\, b_1\left(\frac{1}{6}B^3 + \frac{2}{45\pi}\mu^3\right) - \frac{1}{2}\mu\, \lambda_d^2 \\
&\qquad - \frac{1}{2}\mu\, \lambda_d\, \vartheta_0\left(B - \frac{4}{3\pi}\mu\right) \\
&\qquad - \frac{1}{4}\mu\, \lambda_d\, \vartheta_1\left(B^2 - \frac{1}{4}\mu^2\right)\Big\}\Big] \quad \ldots\ldots \quad (32)
\end{aligned}$$

2. Nontwisted, Tapered Blade

The blade chord is to be a function of the radius of the form

$$t = t_0\,(1 + p \cdot x) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (33)$$

Flapping motion.- The coefficients a_0, a_1, b_1 of the flapping motion are as usual obtained by Fourier division of the thrust moment M_s.

$$\begin{aligned} M_s &= \int_0^{BR} v^2 \cdot r \cdot t \cdot c_a \cdot \varrho/2\, dr \\ &= U^2 \cdot R^2 \cdot t_0 \cdot c_a' \cdot \varrho/2 \left[\int_0^B v_x^2 \cdot x \cdot \left(\frac{v_y}{v_x} + \vartheta_0\right)(1 + p\,x)\,dx \right. \\ &\quad \left. - 2 \int_0^{-\mu \sin\psi} v_x^2 \cdot x \left(\frac{v_y}{v_x} + \vartheta_0\right)(1 + p\,x)\,dx \right]_\pi^{2\pi} \quad . \; . \; . \; . \quad (34) \end{aligned}$$

The manner of writing $\Big]_\pi^{2\pi}$ indicates that the second expression should be allowed for only for the region of the returning blade ($\psi = \pi \div 2\pi$). From equation (34) follows:

$$\begin{aligned} \frac{M_s}{\omega^2 R^4 \cdot t_0 \cdot c_a' \cdot \varrho/2} &= \int_0^B v_x \cdot v_y \cdot x \cdot dx + \vartheta_0 \int_0^B v_x^2 \cdot x \cdot dx \\ &\quad + p \int_0^B v_x \;\; v_y \cdot x^2 \cdot dx + p\,\vartheta_0 \int_0^B v_x^2 \cdot x^2 \cdot dx \\ &\quad - 2 \left[\int_0^{-\mu \sin\psi} v_x \cdot v_y \cdot x \cdot dx + \vartheta_0 \int_0^{-\mu \sin\psi} v_x^2 \cdot x \cdot dx \right. \\ &\quad \left. + p \int_0^{-\mu \sin\psi} v_x \cdot v_y \cdot x^2\,dx + p\,\vartheta_0 \int_0^{-\mu \sin\psi} v_x^2 \cdot x^2\,dx \right]_\pi^{2\pi} \\ &\quad . \; . \; . \; . \; . \; . \; . \; . \quad (35) \end{aligned}$$

The harmonic analysis of this expression gives:

$$\begin{aligned} \frac{M_s}{t_0 \cdot c_a' \cdot \varrho/2 \cdot \omega^2 R^4} &= \frac{1}{3}\lambda_d B^3 + 0{,}08\,\mu^3 \lambda_d + \frac{1}{4}\vartheta_0 \left(B^4 + \mu^2 B^2 - \frac{1}{8}\mu^4\right) \\ &\quad + p\left\{\vartheta_0 \left(\frac{1}{5} B^5 + \frac{1}{6}\mu^2 B^3\right) + \frac{1}{4} B^4 \lambda_d - \frac{3}{32}\mu^4 \lambda_d\right\} \\ &\quad + \sin\psi \left\{\frac{2}{3}\mu\,\vartheta_0 \cdot B^3 + 0{,}053\,\mu^4 \vartheta_0 - \frac{1}{4} a_1 \cdot B^4 \right. \\ &\quad + \frac{1}{2}\mu\,\lambda_d \cdot B^2 - \frac{1}{8}\mu^3 \lambda_d + \frac{1}{8}\mu^2 a_1 B^2 \\ &\quad + p\left(\frac{1}{2}\mu B^4 \vartheta_0 + \frac{1}{3} B^3 \mu \lambda_d \right. \\ &\quad \left.\left. - \frac{1}{5} B^5 a_1 + \frac{1}{12} B^3 \mu^2 a_1\right)\right\} \\ &\quad + \cos\psi \left\{-\frac{1}{3}\mu\,a_0 B^3 - 0{,}035\,\mu^4 a_0 \right. \\ &\quad + \frac{1}{4} b_1 B^4 + \frac{1}{8}\mu^2 b_1 B^2 \\ &\quad + p\left(\frac{1}{5} B^5 b_1 - \frac{1}{4} B^4 \mu\,a_0 \right. \\ &\quad \left.\left. + \frac{1}{12} B^3 \mu^2 b_1\right)\right\} \quad . \; . \; . \; . \quad (36) \end{aligned}$$

Since in this Fourier series the constant term with the exception of one factor must be equal to a_0 and the factors of $\sin\psi$ and $\cos\psi$ must equal zero, the coefficients of the flapping motion are as follows:

$$a_0 = \frac{1}{2}\gamma\left[\frac{1}{3}\lambda_d B^3 + 0{,}08\,\mu^3\lambda_d + \frac{1}{4}\vartheta_0\left(B^4 + \mu^2 B^2 - \frac{1}{8}\mu^4\right)\right.$$
$$\left. + p\left\{\frac{1}{4}\lambda_d B^4 - \frac{3}{32}\mu^4\lambda_d + \vartheta_0\left(\frac{1}{5}B^5 + \frac{1}{6}\mu^2 B^3\right)\right\}\right] - \frac{M_G}{I\cdot\omega^2} \quad \ldots\ldots\ldots (37)$$

$$a_1 = \frac{2\mu}{B^4 - \frac{1}{2}\mu^2 B^2 + p\,B^3\left(0{,}8\,B^2 - \frac{1}{3}\mu^2\right)}$$
$$\times\left\{\lambda_d\left(B^2 - \frac{1}{4}\mu^2\right) + \vartheta_0\left(\frac{4}{3}B^3 + 0{,}106\,\mu^3\right)\right.$$
$$\left. + p\left(\vartheta_0 B^4 + \frac{2}{3}B^3\lambda_d\right)\right\} \quad \ldots\ldots (38)$$

$$b_1 = \frac{4\mu a_0 B\left(\frac{1}{3} + \frac{1}{4}B\cdot p + \frac{0{,}035}{B^3}\cdot\mu^3\right)}{B^2 + \frac{1}{2}\mu^2 + p\left(0{,}8\,B^3 + \frac{1}{3}\mu^2 B\right)} \quad \ldots\ldots (39)$$

Axial thrust.- The supplementary axial thrust in relation to the rectangular blade with chord t_0, is:

$$\Delta k_{sa} = \frac{p\cdot\sigma\cdot c_a'}{2\pi}\left[\int_0^B\int_0^\pi v_x^2\cdot x\left(\frac{v_y}{v_x} + \vartheta_0\right)dx\,d\psi\right.$$
$$+ \int_{-\mu\sin\psi}^{B}\int_{\pi}^{2\pi} v_x^2\cdot x\left(\frac{v_y}{v_x} + \vartheta_0\right)dx\,d\psi$$
$$\left. + \int_0^{-\mu\sin\psi}\int_\pi^{2\pi} v_x^2\cdot x\left(-\frac{v_y}{v_x} - \vartheta_0\right)dx\,d\psi\right]$$
$$= \frac{p\cdot\sigma\cdot c_a'}{2\pi}\left[\iint^x v_x\cdot v_y\cdot x\,dx\,d\psi + \vartheta_0\iint^x v_x^2\cdot x\,dx\,d\psi\right] \quad (40)$$

In conjunction with the axial thrust for the rectangular blade according to equation (18), it therefore gives for the nontwisted, tapered blade

$$k_{sa_t} = \sigma\,c_a'\left\{\frac{1}{2}\lambda_d\left(B^2 + \frac{1}{2}\mu^2\right) + \frac{1}{8}\mu^3 a_1\right.$$
$$+ \vartheta_0\left(\frac{1}{3}B^3 + \frac{1}{2}\mu^2 B - \frac{4}{9\pi}\mu^3\right)$$
$$+ p\,\lambda_d\left(\frac{1}{3}B^3 + \frac{2}{9\pi}\mu^3\right)$$
$$\left. + p\,\vartheta_0\left(\frac{1}{4}B^4 + \frac{1}{4}B^2\mu^2 - \frac{1}{32}\mu^4\right)\right\} \quad \ldots (41)$$

Torque.- Here also the additive terms relative to the rectangular blade are considered. The share of c_a in analogy with the consideration of equation (21) is:

$$\Delta k_{d\,ca} = -\frac{p\cdot\sigma\cdot c_a'}{2\pi}\left[\iint^x v_y^2\cdot x^2\cdot dx\,d\psi\right.$$
$$\left. + \vartheta_0\iint^x v_x\cdot v_y\cdot x^2\,dx\,d\psi\right] \quad (42)$$

and the additive decelerating moment from c_w is, according to equation (24):

$$\Delta k_{d\,c_w} = \frac{\rho \cdot \sigma}{2\pi}\Big[(c_0 + c_1\cdot\vartheta_0 + c_2\cdot\vartheta_0^2)\iint v_x^2\cdot x^2\cdot dx\,d\psi + c_2\iint v_y^2\cdot x^2\cdot dx\cdot d\psi + (c_1 + 2\,\vartheta_0\,c_2)\int_0^1\int_0^{2\pi} v_x\cdot v_y\cdot x^2\cdot dx\,d\psi\Big] \quad (43)$$

After combining and arranging the terms, the final result for the torque of a nontwisted, tapered blade is:

$$\begin{aligned}
\frac{k_{dt}}{\sigma} = {} & (c_0 + \vartheta_0\cdot c_1 + \vartheta_0^2\cdot c_2)\left(\frac{1}{4} + \frac{1}{4}\mu^2 - \frac{1}{32}\mu^4\right) \\
& + (c_1 + 2\,\vartheta_0\cdot c_2)\cdot\frac{1}{3}\lambda_d \\
& + c_2\Big\{a_1^2\left(\frac{1}{8} + \frac{3}{16}\mu^2\right) + \mu\,\lambda_d\,a_1\left(\frac{1}{2} - \frac{3}{8}\mu^2\right) \\
& \quad + \lambda_d^2\left(\frac{1}{2} - \frac{1}{4}\mu^2\right) - \frac{1}{3}\mu\,a_0\,b_1 + a_0^2\left(\frac{1}{4}\mu^2 - \frac{1}{16}\mu^4\right) \\
& \quad + b_1^2\left(\frac{1}{8} + \frac{1}{16}\mu^2\right)\Big\} \\
& - c_a'\Big\{\lambda_d^2\left(\frac{1}{2}B^2 - \frac{1}{4}\mu^2\right) + \lambda_d\,\vartheta_0\left(\frac{1}{3}B^3 + \frac{2}{9\pi}\mu^3\right) \\
& \quad + \mu\,\lambda_d\,a_1\left(\frac{1}{2}B^2 - \frac{3}{8}\mu^2\right) + a_0^2\left(\frac{1}{4}\mu^2 B^2 - \frac{1}{16}\mu^4\right) \\
& \quad - \frac{1}{3}\mu\,a_0\,b_1\cdot B^3 + a_1^2\left(\frac{1}{8}B^4 + \frac{3}{16}\mu^2 B^2\right) \\
& \quad + b_1^2\left(\frac{1}{8}B^4 + \frac{1}{16}\mu^2 B^2\right)\Big\} \\
& + p\Big[(c_0 + c_1\cdot\vartheta_0 + c_2\cdot\vartheta_0^2)\left(\frac{1}{5} + \frac{1}{6}\mu^2\right) + \frac{1}{4}\lambda_d\left(c_1 + 2\,\vartheta_0\,c_2\right) \\
& \quad + c_2\Big\{a_1^2\left(\frac{1}{10} + \frac{1}{8}\mu^2\right) + \mu\,\lambda_d\,a_1\left(\frac{1}{3} - \frac{32}{45\pi}\mu^3\right) \\
& \quad + \lambda_d^2\left(\frac{1}{3} - \frac{4}{9\mu}\mu^3\right) - \frac{1}{4}\mu\,a_0\,b_1 + \frac{1}{6}\mu^2 a_0^2 \\
& \quad + b_1^2\left(\frac{1}{10} + \frac{1}{24}\mu^2\right)\Big\} \\
& \quad - c_a'\Big\{\lambda_d^2\left(\frac{1}{3}B^3 - \frac{4}{9\pi}\mu^3\right) + \lambda_d\,\vartheta_0\left(\frac{1}{4}B^4 + \frac{1}{32}\mu^4\right) \\
& \quad + \mu\,\lambda_d\,a_1\left(\frac{1}{3}B^3 - \frac{32}{45\pi}\mu^3\right) + a_0^2\cdot\frac{1}{6}B^3\mu^2 \\
& \quad - \frac{1}{4}\mu\,a_0\,b_1\cdot B^4 + a_1^2\left(\frac{1}{10}B^5 + \frac{1}{8}B^3\mu^2\right) \\
& \quad + b_1^2\left(\frac{1}{10}B^5 + \frac{1}{24}\mu^2 B^3\right)\Big\}\Big] \quad \ldots\ldots\ldots (44)
\end{aligned}$$

<u>Normal thrust</u>.- The share of c_w in the normal plane is, conformable to equation (27):

$$\Delta k_{sn_I} = \frac{p\cdot\sigma}{2\pi}\Big[(c_0 + c_1\cdot\vartheta_0 + c_2\cdot\vartheta_0^2)\iint v_x^2\cdot x\cdot\sin\psi\,dx\,d\psi + c_2\iint v_y^2\cdot x\cdot\sin\psi\cdot dx\,d\psi + (c_1 + 2\,\vartheta_0\cdot c_2)\int_0^1\int_0^{2\pi} v_x\cdot v_y\cdot x\cdot\sin\psi\,dx\,d\psi\Big] \quad (45)$$

and the share of c_a in the normal plane, according to equation (29):

$$\Delta k_{sn_{II}} = -\frac{p \cdot \sigma \cdot c_a'}{2\pi}\Big[\iint^x v_y^2 \cdot x \cdot \sin\psi \, dx \, d\psi + \vartheta_0 \iint^x v_x \cdot v_y \cdot x \cdot \sin\psi \, dx \, d\psi\Big] \quad . \; . \; . \; . \quad (46)$$

The increment from the lift component is in accord with equation (31):

$$\begin{aligned}\Delta k_{sn_{III}} = -\frac{p \cdot \sigma \cdot c_a'}{2\pi}\Big[&+ a_0 \iint^x v_x \cdot v_y \cdot x \cdot \cos\psi \, dx \, d\psi \\ &- a_1 \iint^x v_x \cdot v_y \cdot x \cdot \cos^2\psi \, dx \, d\psi \\ &- b_1 \iint^x v_x \cdot v_y \cdot x \cdot \cos\psi \sin\psi \, dx \, d\psi \\ &+ \vartheta_0 a_0 \iint^x v_x^2 \cdot x \cdot \cos\psi \, dx \, d\psi \\ &- \vartheta_0 a_1 \iint^x v_x^2 \cdot x \cdot \cos^2\psi \, dx \, d\psi \\ &- \vartheta_0 b_1 \iint^x v_x^2 \cdot x \cdot \cos\psi \cdot \sin\psi \, dx \, d\psi\Big] \quad (47)\end{aligned}$$

By combining these terms with equation (32), the normal thrust of a tapered blade follows as

$$\begin{aligned}\frac{k_{snt}}{\sigma} = &(c_0 + c_1 \cdot \vartheta_0 + c_2 \cdot \vartheta_0^2)\left(\frac{1}{2}\mu + \frac{1}{8}\mu^3\right) \\ &+ (2 c_2 \vartheta_0 + c_1)\left(\frac{1}{2}\mu\lambda_d - \frac{1}{6}a_1 + \frac{1}{8}\mu^2 a_1\right) \\ &+ c_2\Big\{\frac{1}{2}\mu\lambda_d^2 + \frac{1}{8}\mu^3 a_0^2 - a_0 b_1\left(\frac{1}{4}\mu^2 - \frac{4}{15\pi}\mu^3\right) \\ &\quad - \lambda_d a_1\left(\frac{1}{2} - \frac{5}{8}\mu^2\right) - a_1^2\left(\frac{1}{8}\mu - \frac{11}{48}\mu^3\right) \\ &\quad + b_1^2\left(\frac{1}{8}\mu + \frac{1}{48}\mu^3\right)\Big\} \\ &+ c_a'\Big\{a_1 \cdot \lambda_d\left(\frac{3}{4}B^2 - \frac{9}{16}\mu^2\right) + a_1 \cdot \vartheta_0\left(\frac{1}{3}B^3 + \frac{10}{45\pi}\mu^3\right) \\ &\quad + a_0^2\left(\frac{1}{4}B^2\mu - \frac{1}{16}\mu^3\right) + a_1^2\left(\frac{1}{4}B^2\mu - \frac{3}{16}\mu^3\right) \\ &\quad - a_0 b_1\left(\frac{1}{6}B^3 + \frac{2}{45\pi}\mu^3\right) - \frac{1}{2}\mu\lambda_d^2 \\ &\quad - \frac{1}{2}\mu\lambda_d \cdot \vartheta_0\left(B - \frac{4}{3\pi}\cdot\mu\right)\Big\} \\ &+ p\Big[(c_0 + c_1 \cdot \vartheta_0 + c_2\vartheta_0^2)\left(\frac{1}{3}\mu + \frac{4}{45\pi}\mu^4\right) \\ &\quad + (2 c_2 \vartheta_0 + c_1)\left(\frac{1}{4}\mu\lambda_d - \frac{1}{8}a_1 + \frac{1}{16}\mu^2 a_1\right) \\ &\quad + c_2\Big\{\frac{2}{3\pi}\mu^2\lambda_d^2 + \frac{2}{15\pi}\mu^4 a_0^2 - a_0 b_1\left(\frac{1}{8}\mu^2 - \frac{1}{48}\mu^4\right) \\ &\quad - \lambda_d a_1\left(\frac{1}{3} - \frac{44}{45\pi}\mu^3\right) - a_1^2\left(\frac{1}{12}\mu - \frac{44}{105\pi}\mu^4\right) \\ &\quad - b_1^2\left(\frac{1}{12}\mu + \frac{4}{315\pi}\mu^4\right)\Big\} \\ &+ c_a'\Big\{a_1\lambda_d\left(\frac{1}{2}B^3 - \frac{14}{15\pi}\mu^3\right) + a_1\vartheta_0\left(\frac{1}{4}B^4 + \frac{1}{32}\mu^4\right) \\ &\quad + a_0^2\left(\frac{1}{6}B^3\mu - \frac{4}{45\pi}\mu^4\right) + a_1^2\left(\frac{1}{6}B^3\mu - \frac{122}{315\pi}\mu^4\right) \\ &\quad - a_0 b_1\left(\frac{1}{8}B^4 + \frac{1}{192}\mu^4\right) - \frac{2}{3\pi}\mu^2\lambda_d^2 \\ &\quad - \frac{1}{4}\mu\lambda_d\vartheta_0\left(B^2 - \frac{1}{4}\mu^2\right)\Big\}\Big] \quad . \; . \; . \; . \; . \; . \quad (48)\end{aligned}$$

V. CORRECTIONS

1. Allowance for Finite Flapping-Hinge Distance from the Rotor Axis

So far it had been assumed that the flapping hinge of the blade coincided with the axis of rotation. But in the majority of constructions this hinge is a certain distance away from the rotor axis and for such cases the flapping motion and, consequently, the aerodynamic characteristics of a rotor are erroneously reproduced.

If e indicates the nondimensional distance of the flapping hinge, the thrust moment with respect to this hinge is:

$$M_s = \int_{eR}^{BR} v^2 \frac{\rho}{2} c_a \ t \ (x - e) \ R \ dr$$

$$= U^2 \frac{\rho}{2} t \ R^2 \left[\int_e^B c_a \ v_x^2 \ x \ dx - e \int_e^B v_x^2 \ c_a \ dx \right] \qquad (49)$$

Since $e \leqq 0.1$ in the practical designs, the lower integration limit e can be fairly accurately put at 0. Then

$$M_s = R^2 \ U^2 \ t \ \frac{\rho}{2} \left[\int_0^B c_a \ v_x^2 \ x \ dx - e \int_0^B v_x^2 \ c_a \ dx \right] \qquad (50)$$

where the first integral expresses the lift moment conformable to the cited considerations with $e = 0$; the second term comprises the change due to the outwardly shifted flapping hinge.

a) Linearly twisted rectangular blade.- For this moment change the insertion of the substitute function for c_a according to equations (5) and (9) gives:

$$-\frac{\Delta M_s}{R^2 U^2 t_o c_a' \frac{\rho}{2}} = e \int_0^B v_x^2 \left(\frac{v_y}{v_x} + \vartheta_o + x \vartheta_1\right) dx$$

$$-2e \int_0^{-\mu \sin\psi} v_x^2 \left(\frac{v_y}{v_x} + \vartheta_o + x \vartheta_1\right) dx \Bigg]_{\pi}^{2\pi} \qquad (51)$$

where $\Big]_{\pi}^{2\pi}$ again indicates that the second integral is to be considered only for $\psi = \pi$ to 2π. Neglecting this correction, which in fact is unimportant for the final result at the practical coefficients of advance, the harmonic analysis of the expression for the moment change conformable to equation (51) gives:

$$-\frac{\Delta M_s}{\frac{\rho}{2} R^2 U^2 t_o c_a'} = e \left[\frac{1}{2} \lambda_d B^2 + \vartheta_o \left(\frac{1}{3} B^3 + \frac{1}{2} \mu^2 B\right)\right.$$

$$+ \vartheta_1 \left(\frac{1}{4} B^4 + \frac{1}{4} \mu^2 B^2\right)$$

$$+ \sin\psi \left\{\mu \vartheta_o B^2 + \frac{2}{3} \mu \vartheta_1 B^3\right.$$

$$\left. - \frac{1}{3} a_1 B^3 + \mu \lambda_d B + \frac{1}{4} \mu^2 a_1 B\right\}$$

$$+ \cos\psi \left\{-\frac{1}{2} \mu a_o B^2\right.$$

$$\left.\left. + \frac{1}{3} b_1 B^3 + \frac{1}{4} \mu^2 b_1 B\right\}\right] \qquad (52)$$

This affords as new equations for the Fourier coefficients of the flapping moments on a twisted rectangular

$$a_0 = \frac{\gamma}{2}\left[\frac{1}{3}\lambda_d B^3 + 0{,}080\,\mu^3\lambda_d + \frac{1}{4}\vartheta_0\left(B^4 + \mu^2 B^2 - \frac{\mu^4}{8}\right)\right.$$
$$+ \frac{1}{5}\vartheta_1\left(B^5 + \frac{5}{6}\mu^2 B^3\right)$$
$$- e\left\{\frac{1}{2}\lambda_d B^2 + \vartheta_0\left(\frac{1}{3}B^3 + \frac{1}{2}\mu^2 B\right)\right.$$
$$\left.\left. + \vartheta_1\left(\frac{1}{4}B^4 + \frac{1}{4}\mu^2 B^2\right)\right\}\right] - \frac{M_G}{I\cdot\omega^2} \quad . \; . \; . \quad (53)$$

$$a_1 = \frac{2\,\mu}{B^4 - 0{,}5\,\mu^2 B^2 - e\left(\frac{4}{3}B^3 - \mu^2 B\right)} \times \left\{\lambda_d\left(B^2 - \frac{1}{4}\mu^2\right)\right.$$
$$+ \vartheta_0\left(\frac{4}{3}B^3 + 0{,}106\,\mu^3\right) + \vartheta_1\cdot B^4$$
$$\left. - e\left(2\,\lambda_d B + 2\,\vartheta_0 B^2 + \frac{4}{3}\vartheta_1\cdot B^3\right)\right\} \quad . \; . \; . \quad (54)$$

$$b_1 = a_0\,\frac{4\,\mu\,B\left(\frac{1}{3} + \frac{0{,}035}{B^3}\cdot\mu^3 - \frac{e}{2\,B}\right)}{B^2 + \frac{1}{2}\mu^2 - e\left(\frac{4}{3}B + \frac{\mu^2}{B}\right)} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (55)$$

The harmonic analysis of this integral gives:

For equation (56) see page 26

$$-\frac{\Delta M_s}{R^2\,U^2\cdot t_0\cdot c_a'\cdot\varrho/2} = e\left[\frac{1}{2}\lambda_d B^2 + \vartheta_0\left(\frac{1}{3}B^3 + \frac{1}{2}\mu^2 B\right)\right.$$
$$+ p\left\{\lambda_d\left(\frac{1}{3}B^3 + 0{,}08\,\mu^3\right)\right.$$
$$\left. + \vartheta_0\left(\frac{1}{4}B^4 + \frac{1}{4}\mu^2 B^2 - \frac{1}{32}\mu^4\right)\right\}$$
$$+ \sin\psi\left\{\mu\,\vartheta_0 B^2 - \frac{1}{3}a_1 B^3 + \mu\,\lambda_d B + \frac{1}{4}\mu^2 a_1\cdot B\right.$$
$$+ p\left(\frac{2}{3}\mu\,\vartheta_0 B^3 + 0{,}053\,\mu^4\vartheta_0 - \frac{1}{4}a_1 B^4\right.$$
$$\left.\left. + \frac{1}{2}\mu\,\lambda_d B^2 - \frac{1}{8}\mu^3\lambda_d + \frac{1}{8}\mu^2 a_1 B^2\right)\right\}$$
$$+ \cos\psi\left\{-\frac{1}{2}\mu\,a_0 B^2 + \frac{1}{3}b_1\cdot B^3 + \frac{1}{4}\mu^2 b_1 B\right.$$
$$+ p\left(-\frac{1}{3}\mu\,a_0 B^3 - 0{,}035\,\mu^4 a_0\right.$$
$$\left.\left.\left. + \frac{1}{4}b_1 B^4 + \frac{1}{8}\mu^2 b_1 B^2\right)\right\}\right] \quad . \; . \; . \; . \; . \; . \; . \; . \quad (57)$$

The flapping motion coefficients for a tapered blade and a flapping hinge shifted out of the axis of rotation (see footnote, p.26), are then as follows:

$$a_0 = \frac{\gamma}{2}\left[(1 - p\cdot e)\left\{\frac{1}{3}\lambda_d B^3 + 0{,}08\,\mu^3\lambda_d + \frac{1}{4}\vartheta_0\left(B^4 + \mu^2 B^2 - \frac{1}{8}\mu^4\right)\right\}\right.$$
$$+ p\left\{\frac{1}{4}\lambda_d B^4 - 0{,}094\,\mu^4\lambda_d + \vartheta_0\left(\frac{1}{5}B^5 + \frac{1}{6}\mu^2 B^3\right)\right\}$$
$$\left. - e\left\{\frac{1}{2}\lambda_d B^2 + \vartheta_0\left(\frac{1}{3}B^3 + \frac{1}{2}\mu^2 B\right)\right\}\right] - \frac{M_G}{I\cdot\omega^2} \quad . \quad (58)$$

$$a_1 = \frac{2\,\mu}{(B^4 - 0{,}5\,\mu^2 B^2)(1 - pe) + p\left(0{,}8\,B^5 - \frac{1}{3}\mu^2 B^3\right) - e\left(\frac{4}{3}B^3 - \mu^2 B\right)}$$
$$\times\left[\left\{\lambda_d\left(B^2 - \frac{1}{4}\mu^2\right) + \vartheta_0\left(\frac{4}{3}\cdot B^3 + 0{,}106\,\mu^3\right)\right\}(1 - p\cdot e)\right.$$
$$\left. + p\left(B^4\vartheta_0 + \frac{2}{3}B^3\lambda_d\right) - e\,(2\,\vartheta_0 B^2 + 2\,\lambda_d B)\right] \; . \; . \quad (59)$$

$$b_1 = a_0\cdot\frac{4\,\mu\,B\left\{(1 - pe)\left(\frac{1}{3} + \frac{0{,}035}{B^3}\cdot\mu^3\right) + p\cdot\frac{B}{4} - \frac{e}{2\,B}\right\}}{(1 - pe)\left(B^2 + \frac{1}{2}\mu^2\right) + p\left(0{,}8\,B^3 + \frac{1}{3}\mu^2 B\right) - e\left(\frac{4}{3}B + \frac{\mu^2}{B}\right)} \quad (60)$$

blade and a hinge shifted out of the axis of rotation:*

b) Nontwisted, tapered blade.- The equation for the change in lift moment is:

$$- \frac{\Delta M_s}{R^2 U^2 t_o c_a' \rho/2} = e \left[\int_0^B v_x^2 \left(\frac{v_y}{v_x} + \vartheta_o\right) dx \right.$$

$$\left. + p \int_0^B v_x^2 x \left(\frac{v_y}{v_x} + \vartheta_o\right) dx \right]$$

$$= e \left[\int_0^B v_x v_y dx + \vartheta_o \int_0^B v_x^2 dx \right.$$

$$\left. + p \int_0^B v_x v_y x dx + p \vartheta_o \int_0^B v_x^2 x dx \right] \quad (56)$$

---

*Equations (54,55) and (59,60) for defining a_1 and b_1 are the result of the simplified assumption that the moment of the blade mass forces

$$I = \frac{1}{g} \int_{eR}^{R} P_{(r)} r(r-e R) dr$$

equals the mass moment of inertia:

$$I_m = \frac{1}{g} \int_{eR}^{R} P_{(r)} (r-e R)^2 dr$$

The exact solution gives two equations with unknown a_1 and b_1, which are obtained when the nondimensional factor of $\sin \psi$ in the Fourier series for the thrust moment is equated to $\frac{b_1 (I_m - I)}{\frac{\rho}{2} t_o c_a' R^4}$, and that of $\cos \psi$ is equated to

$$\frac{a_1 (I_m - I)}{\frac{\rho}{2} t_o c_a' R^4}.$$

Note: For equations (57) to (60) inclusive, see page 25.

The subsequent calculation of the rotor is effected with the normal formulas, wherein the coefficients of the new flapping motion, according to equations (53) to (55) or (58) to (60) are simply inserted. Strictly speaking, allowance should also be made, when determining the flow angle φ at the blade section, that in the expression for v_y, according to equation (3), the term due to the flapping velocity

$$x \frac{d\beta}{d\psi} \quad \text{in} \quad (x - e) \frac{d\beta}{d\psi}$$

changes, though this can be ignored for values of $e \leqq 0.1$.

2. Change in Profile Drag Coefficient Due to Yaw of the Blade at Coefficients of Advance

In the determination of the resultant flow velocity of a blade section during rotation, it had thus far been assumed that the flow in the normal plane is always perpendicular to the blade - i.e., the velocity component in blade direction was ignored. In reality, the resultant velocity in the normal plane consists of the circumferential velocity of the particular blade element and the flying speed $V = \mu U$ (fig. 5):

$$v_{resultant} = x U \hat{+} \mu U$$

hence the blade section is exposed under a changing angle δ while rotating. It is:

$$\tan \delta = \frac{\mu \cos \psi}{x + \mu \sin \psi} \tag{61}$$

Since the polar of a profile, especially the drag, changes in yaw with the angle of sideslip δ, an average value for this δ is formed as function of the coefficient of advance. In this manner, a comparison with corresponding wind-tunnel tests affords a correction factor for the drag coefficient which takes care of the yaw of the blade at coefficients of advance.

Allowance for the rating of the blade elements in the various settings is carried out in such a way that the individual points of the disk area swept by the rotor are classed according to the dynamic pressure under which the blade element operates. For blade setting ψ, it is:

$$\tan \delta_{mean} = \frac{\int_0^1 \delta \, v_x^2 \, dx}{\int_0^1 v_x^2 \, dx}$$

$$= \frac{\int_0^1 \mu \cos \psi \, (x + \mu \sin \psi) \, dx}{\int_0^1 (x + \mu \sin \psi)^2 \, dx} \tag{62}$$

When forming the integral over ψ, it should be observed that for the particular mean value δ_m the absolute amount without consideration of the sign is of interest. The integral should therefore express only half a rotor. Furthermore, δ changes sign on the returning blade for $x = - \mu \sin \psi$. Herewith, we have:

$$\tan \delta_m = \frac{\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}\int_0^1 \mu \cos\psi \, (x+\mu \sin \psi) d\psi \, dx - 2 \int_{\pi}^{\frac{3}{2}\pi}\int_0^{-\mu \sin\psi} \mu \cos\psi (x+\mu \sin \psi) d\psi \, dx}{\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}\int_0^1 (x + \mu \sin \psi)^2 \, d\psi \, dx}$$

$$\tan \delta_m = \frac{\mu \left(1 + \frac{2}{3} \mu^2\right)}{\pi \left(\frac{1}{3} + \frac{1}{2} \mu^2\right)} \tag{63}$$

From the presentation of this function in figure 6, it is seen that the mean angle of sideslip occurring at coefficients of advance is not insignificant. For $\mu = 0.4$, for instance, it amounts to about 20°.

Allowance for this angle of sideslip is now made in such a way that the normal polar is computed with $i\ c_w$ instead of with $c_w = c_o + c_1\ \alpha_p + c_2\ \alpha_p^2$. The correction factor i is to be taken from corresponding wind-tunnel tests as $f(\delta_m)$.

In earlier aerodynamic studies of rotors, the effective flow velocity of a blade had been expressed by the component of the velocity perpendicular to the blade. Hence the calculation was already made with a reduced dynamic pressure. In the evaluation of tests on airfoils in yaw, the coefficient of the drag must accordingly be referred to the flow perpendicular to the blade.

To my knowledge there are at the present time no acceptable test data of this kind for well-rounded blades such as are employed for rotor blades. For this reason, it is very desirable to make such experiments now, in order that the effect of the yaw, which at high coefficients of advance is certainly of no small influence, may be elucidated.

3. Allowance for Break-away of Flow at Returning Blade with Coefficients of Advance

In the investigations on the representation of the air-force coefficients on the blade element at the beginning of the report, it had already been pointed out that the validity of the preceding considerations hinges upon the central and outer parts of a blade during rotation being subject to small aerodynamic operating angles.

However, inasmuch as these assumptions are no longer always complied with at high coefficients of advance, it is necessary to discuss the operating angles α_p at the different points of the rotor disk more in detail. Confined to the normal flight stages, where the flow through the rotor is approximately in the normal plane, the disk area swept by the blades can be fundamentally divided into three zones of operating angle (fig. 7).

On the side of the returning blade ($\psi = 180°$ to $360°$), the blade elements within the disk area

$$r = -\mu R \sin \psi$$

are in the reversed velocity region.

This is followed by a zone B of separated flow for which the representation of the lift coefficient c_a by $c_a' \alpha_p$ is no longer justified. This critical region can be approximately demarcated from the sound flow with $r = -b R \sin \psi$, where b is left to be determined. It is the distance x at which, for $\psi = 270°$, the value $\alpha_p \cong 15°$ is reached, according to the previous considerations.

For $\psi = 270°$, it is

$$v_y = \lambda_d + x a_1$$

$$v_x = x - \mu$$

and consequently,

$$\alpha_p = \frac{\lambda_d + x a_1}{x - \mu} + \vartheta$$

Solving this equation with respect to x, it affords with $\alpha_p = 0.26$

$$b = \frac{\mu (0.26 - \vartheta) + \lambda_d}{0.26 - a_1 - \vartheta}^{*} \tag{64}$$

For the rest of the principal share A of the disk area the angles α_p are smaller than 15°, which confirms the correctness of the theory of the forces at the blade element.

---

*It is advisable, however, to analyze the working angles α_p of the different blade elements for $\psi = 270°$, so as to gain some opinion about the proper selection of b.

The correction for the separated flow, (the shaded area) in figure 7, is applied in such a way that c_a is replaced for this range by a constant value $c_{ao} \cong 0.5\, c_{a\max}$. Naturally the share already contained in the previous equations for k_{sa}, k_{sn} and k_d must be subtracted again from the assumption $c_a = c_a' \alpha_p$. Quite obviously this kind of correction is possible only if b does not exceed the value B, but which will under normal conditions be rarely the case with the present day customary coefficients of advance.

The signs for the subsequently computed correction terms are so chosen that these expressions become additive to the old coefficients. To illustrate: if $k_{sa\,correct}$ is the coefficient of axial thrust with allowance for the separated flow, and Δk_{sa} the correction, it is;

$$k_{sa\,correct} = k_{sa} + \Delta k_{sa} \quad \ldots\ldots\ldots\ldots (65)$$

a) <u>Linearly twisted rectangular blade</u>, **Axial thrust:**

$$\Delta k_{sav} = \frac{\sigma}{2\pi}\left[c_{a_0} \int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x^2 \cdot d\psi\, dx - c_a' \left\{\int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x^2\left(\frac{v_y}{v_x} + \vartheta_0 + x\vartheta_1\right) d\psi\, dx\right\}\right]$$

$$= \frac{\sigma \cdot c_a'}{2\pi}\left[\left(\frac{c_{a_0}}{c_a'} - \vartheta_0\right)\int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x^2 \cdot d\psi\, dx - \vartheta_1 \int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x^2 \cdot x\, d\psi\, dx - \int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x \cdot v_y \cdot d\psi\, dx\right] \quad (66)$$

With the abbreviated method of writing $\zeta^n = b^n - \mu^n$, the result reads,

$$\Delta k_{sav} = -\sigma \cdot c_a' \left[\lambda_d\left(\frac{1}{8}\zeta^2 - \frac{1}{4}\zeta\mu\right) + \frac{1}{16} a_1 (\zeta^3 - \zeta^2\mu - \zeta\mu^2) + \left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\cdot\frac{2}{3\pi}\left(\frac{1}{3}\zeta^3 - \zeta^2\mu + \zeta\mu^2\right) + \vartheta_1\left(\frac{3}{64}\zeta^4 - \frac{1}{8}\zeta^3\mu + \frac{3}{32}\zeta^2\mu^2\right)\right] \quad . (67)$$

<u>Normal thrust</u>.- Similarly, it is for the normal thrust

$$\Delta k_{snv} = \frac{\sigma}{2\pi}\left[-c_{a_0}\int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x^2 \cdot \sin\varphi \sin\psi\, d\psi\, dx + c_a' \int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x^2\left(\frac{v_y}{v_x} + \vartheta_0 + x\vartheta_1\right)\sin\varphi\sin\psi\, d\psi\, dx\right]$$

$$= \frac{\sigma}{2\pi}\left[c_a' \left\{\int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_y^2 \cdot \sin\psi\, d\psi\, dx + \int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} \vartheta_0 \cdot v_x \cdot v_y \cdot \sin\psi\, d\psi\, dx + \int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} \vartheta_1 \cdot v_x \cdot v_y \cdot x \cdot \sin\psi\, d\psi\, dx\right\} - c_{a_0} \cdot \int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x \cdot v_y \cdot \sin\psi\, d\psi\, dx\right] \ldots\ldots (68)$$

This contains only the shares of the forces in the normal plane. The correction from the lift component due to flapping angle may be neglected, since this effect appears at the decisive blade settings of $\psi \cong 270°$ primarily as lateral force, while the reaction on the normal thrust is quite small. On these assumptions the normal thrust correction reads,

$$\begin{aligned}\Delta k_{snv} = & -\sigma \cdot c_a' \Big[\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\Big\{\lambda_d\left(\frac{1}{3\pi}\zeta^2 - \frac{2}{3\pi}\zeta\mu\right) \\ & + a_1\left(\frac{8}{45\pi}\zeta^3 - \frac{1}{5\pi}\zeta^2\mu - \frac{2}{15\pi}\zeta\mu^2\right)\Big\} + \vartheta_1\Big\{\lambda_d\Big(\frac{1}{16}\zeta^3 \\ & - \frac{3}{32}\zeta^2\mu\Big) + a_1\left(\frac{5}{128}\zeta^4 - \frac{1}{24}\zeta^3\mu - \frac{1}{64}\zeta^2\mu^2\right)\Big\} \\ & + \lambda_d a_1\left(\frac{3}{16}\zeta^2 + \frac{1}{8}\zeta\mu\right) + \frac{1}{4}\zeta\lambda_d^2 + a_0^2 \cdot \frac{1}{16}\zeta\mu^2 \\ & + a_1^2\left(\frac{5}{96}\zeta^3 + \frac{1}{32}\zeta^2\mu + \frac{1}{32}\zeta\mu^2\right) \\ & + b_1^2\left(\frac{1}{96}\zeta^3 - \frac{1}{32}\zeta^2\mu + \frac{1}{32}\zeta\mu^2\right) \\ & - a_0 b_1\left(\frac{2}{15\pi}\zeta^2\mu - \frac{4}{15\pi}\zeta\mu^2\right)\Big] \quad \ldots\ldots\ldots \quad (69)\end{aligned}$$

Torque.

$$\begin{aligned}\Delta k_{dv} = & \frac{\sigma}{2\pi}\Big[c_a' \int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x^2 \cdot x \cdot \frac{v_y}{v_x}\left(\frac{v_y}{v_x} + \vartheta_0 + x\vartheta_1\right) d\psi\, dx \\ & - c_{a_0}\int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x^2 \cdot x \cdot \frac{v_y}{v_x}\, d\psi\, dx\Big] \\ = & \frac{c_a'\sigma}{2\pi}\Big[\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x \cdot v_y \cdot x \cdot d\psi\, dx \\ & + \vartheta_1\int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x \cdot v_y \cdot x^2\, d\psi\, dx + \int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_y^2 \cdot x \cdot d\psi\, dx\Big] \quad (70)\end{aligned}$$

The solution is:

$$\begin{aligned}\Delta k_{dv} = & \sigma c_a' \Big[\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\Big\{\lambda_d\left(\frac{2}{9\pi}\zeta^3 - \frac{1}{3\pi}\zeta^2\mu\right) + a_1\Big(\frac{2}{15\pi}\zeta^4 \\ & - \frac{2}{15\pi}\zeta^3\mu - \frac{1}{15\pi}\zeta^2\mu^2\Big)\Big\} + \vartheta_1\Big\{\lambda_d\Big(\frac{3}{64}\zeta^4 \\ & - \frac{1}{16}\zeta^3\mu\Big) + a_1\left(\frac{1}{32}\zeta^5 - \frac{1}{32}\zeta^4\mu - \frac{1}{96}\zeta^3\mu^2\right)\Big\} \\ & + \lambda_d a_1\left(\frac{1}{8}\zeta^3 + \frac{1}{16}\zeta^2\mu\right) + \frac{1}{8}\zeta^2\lambda_d^2 + a_1^2\Big(\frac{5}{128}\zeta^4 \\ & + \frac{1}{48}\zeta^3\mu + \frac{1}{64}\zeta^2\mu^2\Big) + \frac{1}{32}\zeta^2\mu^2 a_0^2 + b_1^2\Big(\frac{1}{128}\zeta^4 \\ & - \frac{1}{48}\zeta^3\mu + \frac{1}{64}\zeta^2\mu^2\Big) - a_0 b_1\Big(\frac{4}{45\pi}\zeta^3\mu \\ & - \frac{2}{15\pi}\zeta^2\mu^2\Big)\Big] \quad \ldots\ldots\ldots\ldots\ldots \quad (71)\end{aligned}$$

b) Non-twisted tapered blade.

Axial thrust. Conformably to equation (66) the axial thrust correction reads:

$$\Delta k_{sat} = -\frac{\sigma \cdot c_a'}{2\pi}\left[\int\limits_{\pi}^{2\pi}\int\limits_{-\mu\sin\psi}^{-b\sin\psi}\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right) v_x^2\, d\psi\, dx + \int\limits_{\pi}^{2\pi}\int\limits_{-\mu\sin\psi}^{-b\sin\psi} v_x \cdot v_y \cdot d\psi\, dx + p\left\{\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\int\limits_{\pi}^{2\pi}\int\limits_{-\mu\sin\psi}^{-b\sin\psi} v_x^2 \cdot x\, d\psi\, dx + \int\limits_{\pi}^{2\pi}\int\limits_{-\mu\sin\psi}^{-b\sin\psi} v_x \cdot v_y \cdot x\, d\psi\, dx\right\}\right] \quad . . . (72)$$

Which, evaluated, gives;

$$\Delta k_{sat} = -\sigma \cdot c_a'\left[\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\left(\frac{2}{9\pi}\zeta^3 - \frac{2}{3\pi}\zeta^2\mu + \frac{2}{3\pi}\zeta\mu^2\right) + \lambda_d\left(\frac{1}{8}\zeta^2 - \frac{1}{4}\zeta\mu\right) + \frac{1}{16}a_1(\zeta^3 - \zeta^2\mu - \zeta\mu^2) + p\left\{\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\left(\frac{3}{64}\zeta^4 - \frac{1}{8}\zeta^3\mu + \frac{3}{32}\zeta^2\mu^2\right) + \lambda_d\left(\frac{2}{9\pi}\zeta^3 - \frac{1}{3\pi}\zeta^2\mu\right) + \frac{1}{15\pi}a_1(2\zeta^4 - 2\zeta^3\mu - \zeta^2\mu^2)\right\}\right] \quad . . . . (73)$$

Normal thrust. Conformably to equation (68) it is:

$$\Delta k_{snt} = \frac{\sigma \cdot c_a'}{2\pi}\left[\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\int\limits_{\pi}^{2\pi}\int\limits_{-\mu\sin\psi}^{-b\sin\psi} v_x \cdot v_y \cdot \sin\psi\, d\psi\, dx + \int\limits_{\pi}^{2\pi}\int\limits_{-\mu\sin\psi}^{-b\cdot\sin\psi} v_y^2 \cdot \sin\psi\, d\psi\, dx + p\left\{\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\int\limits_{\pi}^{2\pi}\int\limits_{-\mu\sin\psi}^{-b\sin\psi} v_x \cdot v_y \cdot x \cdot \sin\psi\, d\psi\, dx + \int\limits_{\pi}^{2\pi}\int\limits_{-\mu\sin\psi}^{-b\sin\psi} v_y^2 \cdot x \cdot \sin\psi\, d\psi\, dx\right\}\right] \quad . . (74)$$

Hence

$$\Delta k_{snt} = -\sigma \cdot c_a'\left[\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\left\{\lambda_d\left(\frac{1}{3\pi}\zeta^2 - \frac{2}{3\pi}\zeta\mu\right) + a_1\left(\frac{8}{45\pi}\zeta^3 - \frac{1}{5\pi}\zeta^2\mu - \frac{2}{15\pi}\zeta\mu^2\right)\right\} + \lambda_d a_1\left(\frac{3}{16}\zeta^2 + \frac{1}{8}\zeta\mu\right) + \frac{1}{4}\zeta\lambda_d^2 + a_0^2 \cdot \frac{1}{16}\zeta\mu^2 + a_1^2\left(\frac{5}{96}\zeta^3 + \frac{1}{32}\zeta^2\mu + \frac{1}{32}\zeta\mu^2\right) + b_1^2\left(\frac{1}{96}\zeta^3 - \frac{1}{32}\zeta^2\mu + \frac{1}{32}\zeta\mu^2\right) - a_0 b_1\left(\frac{2}{15\pi}\zeta^2\mu - \frac{4}{15\pi}\zeta\mu^2\right)\right.$$

$$
\begin{aligned}
&+ p\Bigg\{\Bigg\{\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\left\{\lambda_d\left(\frac{1}{16}\,\zeta^3 - \frac{3}{32}\,\zeta^2\,\mu\right)\right. \\
&\qquad \left. + a_1\left(\frac{5}{128}\,\zeta^4 - \frac{1}{24}\,\zeta^3\,\mu - \frac{1}{64}\,\zeta^2\,\mu^2\right)\right\} \\
&\qquad + \lambda_d\,a_1\left(\frac{16}{45\pi}\,\zeta^3 + \frac{2}{15\pi}\,\zeta^2\,\mu\right) \\
&\qquad + a_0^2\cdot\frac{1}{15\pi}\,\zeta^2\,\mu^2 + \frac{1}{3\pi}\,\zeta^2\,\lambda_d^2 \\
&\qquad + a_1^2\left(\frac{4}{35\pi}\,\zeta^4 + \frac{16}{315\pi}\,\zeta^3\,\mu + \frac{1}{35\pi}\,\zeta^2\,\mu^2\right) \\
&\qquad + b_1^2\left(\frac{2}{105\pi}\,\zeta^4 - \frac{16}{315\pi}\,\zeta^3\,\mu + \frac{4}{105\pi}\,\zeta^2\,\mu^2\right) \\
&\qquad - a_0\,b_1\left(\frac{1}{48}\,\zeta^3\,\mu - \frac{1}{32}\,\zeta^2\,\mu^2\right)\Bigg\}\Bigg\}\Bigg] \quad . \; . \; . \quad (75)
\end{aligned}
$$

__Torque__. Conformably to equation (70) the torque correction reads;

$$
\begin{aligned}
\Delta k_{dt} = \frac{\sigma\cdot c_a'}{2\pi}\Bigg[&\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x\cdot v_y\cdot x\,d\,\psi\,dx \\
&+ \int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_y^2\cdot x\cdot d\,\psi\,dx \\
&+ p\left\{\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_x\cdot v_y\cdot x^2\,d\,\psi\,dx\right. \\
&\qquad \left. + \int_{\pi}^{2\pi}\int_{-\mu\sin\psi}^{-b\sin\psi} v_y^2\cdot x^2\,d\,\psi\,d\,x\right\}\Bigg] \quad . \; . \; . \; . \; . \quad (76)
\end{aligned}
$$

The result is;

$$
\begin{aligned}
\Delta k_{dt} = \sigma\cdot c_a'\Bigg[&\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\left\{\lambda_d\left(\frac{2}{9\pi}\,\zeta^3 - \frac{1}{3\pi}\,\zeta^2\,\mu\right) + a_1\left(\frac{2}{15\pi}\,\zeta^4\right.\right. \\
&\left.\left. - \frac{2}{15\pi}\zeta^3\,\mu - \frac{1}{15\pi}\zeta^2\,\mu^2\right)\right\} + \lambda_d\,a_1\left(\frac{1}{8}\,\zeta^3 + \frac{1}{16}\zeta^2\,\mu\right) \\
&+ \frac{1}{8}\,\zeta^2\,\lambda_d^2 + a_1^2\left(\frac{5}{128}\,\zeta^4 + \frac{1}{48}\,\zeta^3\,\mu + \frac{1}{64}\,\zeta^2\,\mu^2\right) \\
&+ \frac{1}{32}\,\zeta^2\,\mu^2\,a_0^2 - a_0\,b_1\left(\frac{4}{45\pi}\,\zeta^3\,\mu - \frac{2}{15\pi}\,\zeta^2\,\mu^2\right) \\
&+ b_1^2\left(\frac{1}{128}\,\xi^4 - \frac{1}{48}\,\zeta^3\,\mu + \frac{1}{64}\,\zeta^2\,\mu^2\right)\Bigg] \\
+ p\,\sigma\cdot c_a'\Bigg[&\left(\vartheta_0 - \frac{c_{a_0}}{c_a'}\right)\left\{\lambda_d\left(\frac{3}{64}\,\zeta^4 - \frac{1}{16}\,\zeta^3\,\mu\right) + a_1\left(\frac{1}{32}\,\zeta^5\right.\right. \\
&\left.\left. - \frac{1}{32}\zeta^4\,\mu - \frac{1}{96}\zeta^3\,\mu^2\right)\right\} + \lambda_d\,a_1\left(\frac{4}{15\pi}\,\zeta^4\right. \\
&\left. + \frac{4}{45\pi}\,\zeta^3\,\mu\right) + \frac{2}{9\pi}\,\zeta^3\,\lambda_d^2 + a_1^2\left(\frac{16}{175\pi}\,\zeta^5\right. \\
&\left. + \frac{4}{105\pi}\,\zeta^4\,\mu + \frac{2}{105\pi}\,\zeta^3\,\mu^2\right) + \frac{2}{45\pi}\,\zeta^3\,\mu^2\,a_0^2 \\
&- a_0\,b_1\left(\frac{1}{64}\,\zeta^4\,\mu - \frac{1}{48}\,\zeta^3\,\mu^2\right) \\
&+ b_1^2\left(\frac{8}{525\pi}\,\zeta^5 - \frac{4}{105\pi}\,\zeta^4\,\mu + \frac{8}{315\pi}\,\zeta^3\,\mu^2\right)\Bigg] \quad (77)
\end{aligned}
$$

REFERENCES AND BIBLIOGRAPHY

1. Glauert, H.: A General Theory of the Autogyro. R. & M. No. 1111, British A.R.C., 1928.

2. Lock, C. N. H.: Further Development of Autogyro Theory. Parts I and II. R. & M. No. 1127, British A.R.C., 1927.

3. Glauert, H.: On the Vertical Ascent of a Helicopter. R. & M. No. 1132, British A.R.C., 1928.

4. Lock, C. N. H., and Townend, H. C. H.: Wind Tunnel Experiments on a Model Autogyro at Small Angles of Incidence. R. & M. No. 1154, British A.R.C., 1928.

5. Glauert, H.: On the Horizontal Flight of a Helicopter. R. & M. No. 1157, British A.R.C., 1928.

6. Squire, H. B.: The Flight of a Helicopter. R. & M. No. 1730, British A.R.C., 1936.

7. Beavan, J. A., and Lock, C. N. H.: The Effect of Blade Twist on the Characteristics of the C.30 Autogiro. R. & M. No. 1727, British A.R.C., 1936.

8. Wheatley, John B.: Lift and Drag Characteristics and Gliding Performance of an Autogiro as Determined in Flight. T.R. No. 434, N.A.C.A., 1932.

9. Wheatley, John B.: Wing Pressure Distribution and Rotor-Blade Motion of an Autogiro as Determined in Flight. T.R. No. 475, N.A.C.A., 1933.

10. Wheatley, John B.: An Aerodynamic Analysis of the Autogiro Rotor with a Comparison between Calculated and Experimental Results. T.R. No. 487, N.A.C.A., 1934.

11. Wheatley, John B., and Hood, Manley J.: Full-Scale Wind-Tunnel Tests of a PCA-2 Autogiro Rotor. T.R. No. 515, N.A.C.A., 1935.

12. Wheatley, John B.: The Influence of Wing Setting on the Wing Load and Rotor Speed of a PCA-2 Autogiro as Determined in Flight. T.R. No. 523, N.A.C.A., 1935.

13. Wheatley, John B., and Bioletti, Carlton: Wind-Tunnel Tests of 10-Foot-Diameter Autogiro Rotors. T.R. No. 552, N.A.C.A., 1936.

14. Wheatley, John B.: An Analytical and Experimental Study of the Effect of Periodic Blade Twist on the Thrust, Torque, and Flapping Motion of an Autogiro Rotor. T.R. No. 591, N.A.C.A., 1937.

15. Wheatley, John B.: An Analysis of the Factors that Determine the Periodic Twist of an Autogiro Rotor Blade, with a Comparison of Predicted and Measured Results. T.R. No. 600, N.A.C.A., 1937.

16. Wheatley, John B.: A Study of Autogiro Rotor-Blade Oscillations in the Plane of the Rotor Disk. T.N. No. 581, N.A.C.A., 1936.

17. Wheatley, John B., and Bioletti, Carlton: Analysis and Model Tests of Autogiro Jump Take-Off. T.N. No. 582, N.A.C.A., 1936.

18. Lamé, Maurice Luc Valere: Le vol vertical; theorie generale des helicopteres, les appareils a voilures tournantes de leurs origines a 1934. Blondel la Rougery, Paris, 1934.

19. Isacco, Vittorio: Modern Helicopter Theory. A Complete Exposition of Up-to-Date Ideas on the Principles of Direct Lift Flight. Aircraft Engineering, vol. 8, October 1936, p. 274.

20. Bréguet, Louis: Les Gyroplanes, Leurs Possibilités de Vitesse et de Rayon d'Action. La Science Aerienne, vol. 5, Nov.-Dec., 1936, p. 327.

21. Bréguet, Louis: The Gyroplane - Its Principles and Its Possibilities. T.M. No. 816, N.A.C.A., 1937.

22. Flachsbart, O.: Theorie der Hubschraube. Z.F.M., Bd. 19, 1928, p. 177.

23. Flachsbart, O., and Kroeber, G.: Experimental Investigation of Aircraft Propellers Exposed to Oblique Air Currents. T.M. No. 562, N.A.C.A., 1930.

24. Schrenk, Martin: Aerodynamic Principles of the Direct Lifting Propeller. T.M. No. 733, N.A.C.A., 1934.

25. Mohring, Th.: Fragen des Drehflüglers. Luftwissen, Bd. 3, 1936, p. 208; Bd. 4, 1937, p. 12.

26. Schoppe, G: Die Gleitzahl des Tragschraubers. Luftwissen, Bd. 3, 1936, p. 216.

27. Walchner, O.: Berechnung von Luftschrauben mit kleinem Schubbeiwert und kleinem Fortschrittsgrad. Luftfahrtforschung, Bd. 13, April 20, 1936, p. 103.

28. Küssner, Hans Georg: Helicopter Problems. T.M. No. 827, N.A.C.A., 1937.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

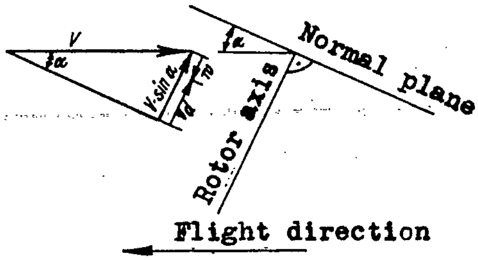


Figure 1.- Axial flow through normal plane.

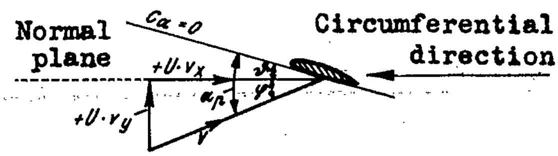
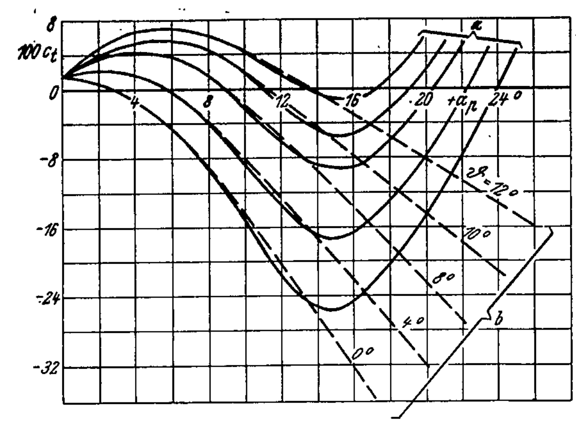


Figure 2.- Velocities and angles at a section of the blade.

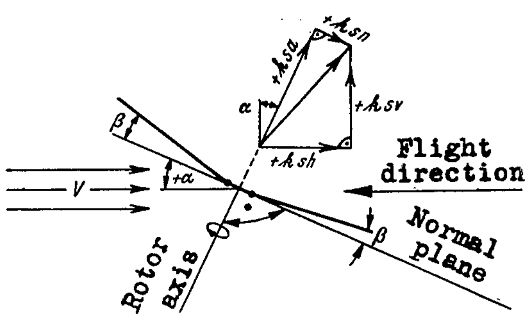


a, Actual variation

b, Variation of substitute function

Figure 3.- Tangential force coefficient as $f(\alpha_p)$ for various blade angles of attack ϑ.

Airfoil section: Göttingen 367



$$k_{sv} = k_{sa} \cos \alpha - k_{sn} \sin \alpha$$
$$k_{sh} = k_{sa} \sin \alpha + k_{sn} \cos \alpha$$

Figure 4.- Air forces and rotor setting.

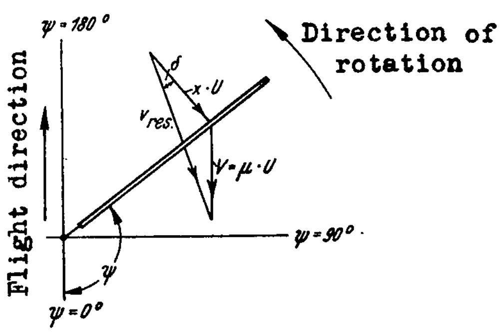


Figure 5.- Blade section in yaw.

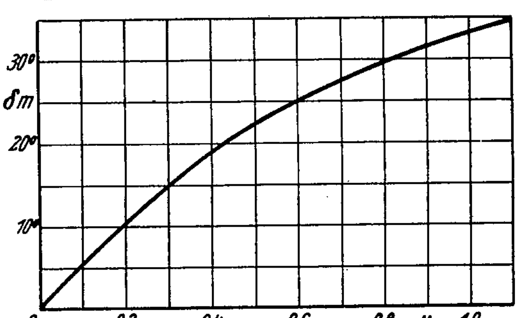
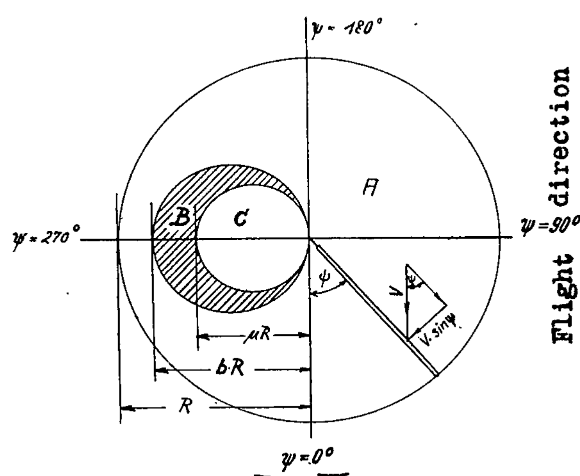


Figure 6.- Mean angle of sideslip plotted against coefficient of advance.



Direction of rotation of rotors

A, Region of sound flow

B, Region of separated flow (idealized)

C, Reversed-velocity region

Figure 7.- Operating angle of blade sections in rotor-swept-disk area.

DO NOT REMOVE SLIP FROM MATERIAL

Delete your name from this slip when returning material to the library.

NAME	MS

NASA Langley (Rev. May 1988) RIAD N-75